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**Note:** After clicking on a link, right click and select “Previous View” to go back to original text.
Welcome to the GACE Study Companion

Get Ready to Show What You Know

You have gained the knowledge and skills you need for your teaching career. Now you are ready to demonstrate your abilities by taking the Georgia Assessments for the Certification of Educators® (GACE®).

Using the GACE Study Companion is a smart way to prepare for the test so you can do your best on test day. The Study Companion can help you stay on track and make the most efficient use of your study time.

The Study Companion contains practical information and helpful tools including:

- An overview of the assessments
- Descriptions of different types of test questions
- Information about scores and how to understand them
- Test-taking tips and strategies
- Frequently asked questions
- Specific information about the assessment you are taking
- Practice questions and explanations of correct answers
- A study plan template
- Additional resources to help you study

So where should you start?

Begin by reviewing the Study Companion in its entirety, paying particular attention to the content specifications in About the Assessment beginning on page 26. The content specifications detail the knowledge and skills to be measured on the assessment. These specifications are aligned with the:

- Common Core Georgia Performance Standards (CCGPS) — these standards are available on the Georgia Department of Education website at www.doe.k12.ga.us under Curriculum and Instruction
- Content standards for Georgia’s state-approved educator preparation programs — see Educator Preparation Rules in the Rules section on the Georgia Professional Standards Commission (GaPSC) website at www.gapsc.com

To identify the areas you may need to study, go through the standards and note the specific areas that you need to review.
Once you have reviewed the Study Companion and the standards, you can create your own personalized study plan and schedule based on your individual needs and how much time you have before test day. Be sure to also seek other resources to strengthen your content knowledge. See the Preparation Resources section of this Study Companion.

Keep in mind that study habits are individual. There are many different ways to successfully prepare for your test. Some people study better on their own, while others prefer a group setting. You may have more energy early in the day, but another test taker may concentrate better in the evening. Use the Study Companion to develop the approach that works best for you.

Your teaching career begins with preparation. Good luck!
Know What to Expect

These questions and answers will give you an overview of the GACE assessments.

Why do I need to take a GACE assessment?

GACE is the educator certification assessment program for the state of Georgia. The purpose of the GACE assessments is to help the Georgia Professional Standards Commission (GaPSC) ensure that candidates have the knowledge and skills needed to perform the job of an educator in Georgia's public schools. Professionals serving in most public schools must hold a valid certificate, appropriate to their field of employment.

Which assessments should I take?

Before you register for an assessment, identify which assessment(s) you need to take. Note that some assessments contain more than one test, and that you will be required to pass all of the tests within an assessment to meet the certification requirements. Educator testing requirements for Georgia are available from the GaPSC website at www.gapsc.com under Educator Assessment.

How do I find out what is covered on the assessment I need to take?

Each Study Companion contains the content specifications for the assessment that detail the knowledge and skills to be covered. These specifications are aligned with the Common Core Georgia Performance Standards and the content standards for Georgia’s state-approved educator preparation programs.

What are the Common Core Georgia Performance Standards (CCGPS)?

The CCGPS is a set of core standards for kindergarten through high school in English language arts, mathematics, and grades 6-12 literacy in science, history, social studies, and technical subjects that have been formally adopted by Georgia and 44 other states, including the District of Columbia (D.C.), and two territories, along with the Department of Defense Education Activity. The CCGPS provide a consistent framework to prepare students for success in college and the 21st century workplace.

What are the content standards for Georgia’s state-approved educator preparation programs?

A set of content standards has been adopted by the GaPSC on which state-approved educator preparation program providers prepare their candidates. These standards are adapted from national content standards.

How are the GACE assessments administered?

All GACE assessments are administered via computer at specially equipped test centers throughout Georgia, in neighboring states, and internationally.
Where and when are the GACE assessments offered?
Test dates and test center locations are available on the GACE website at www.gace.ets.org. When you register for a test, you can select the test center that is most convenient for you.

How do I get my scores?
Your scores are available through your ETS GACE testing account on the GACE website at www.gace.ets.org on the score reporting date listed in the Registration Bulletin. Your test results are released to you, the GaPSC, and your program provider, if you have one.

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
Familiarize Yourself with Test Questions

The GACE assessments include several types of test questions, which can be broken into two categories: **objective items** (for which you select or produce an answer that is scored either correct or incorrect) and **constructed response** (for which you write or record a response of your own that is scored by trained raters based on scoring guidelines). You may be familiar with these question formats from taking other standardized tests. If not, familiarize yourself with them so you don’t spend time during the test figuring out how to answer them.

Understanding Selected-response Questions

The majority of the questions in the GACE assessments are selected-response questions. The single-question format presents a direct question or an incomplete statement. This type of question may begin with the phrase “Which of the following . . .” Take a look at this example:

```
Which of the following is a flavor made from beans?
A. Strawberry
B. Cherry
C. Vanilla
D. Mint
```

How would you answer this question?

All of the answer options are flavors. Your job is to decide which of the flavors is the one made from beans.

Try following these steps to select the correct answer.

1) **Limit your answer to one of the options given.** You may know that chocolate and coffee are also flavors made from beans, but they are not listed. Rather than thinking of other possible answers, focus only on the options given (“Which of the following . . .”).

2) **Eliminate incorrect answers.** You may know that strawberry and cherry flavors are made from fruit and that mint flavor is made from a plant. That leaves vanilla as the only possible answer.

3) **Verify your answer.** You can substitute “vanilla” for the phrase “Which of the following” and turn the question into this statement: “Vanilla is a flavor made from beans.” This will help you be sure that your answer is correct. If you’re still uncertain, try substituting the other options to see if they make sense.

You may want to use this technique as you answer selected-response questions on the practice tests.
**Try a more challenging example**

The vanilla bean question is pretty straightforward, but you'll find that more challenging questions have a similar structure. For example:

Entries in outlines are generally arranged according to which of the following relationships of ideas?

A. Literal and inferential  
B. Concrete and abstract  
C. Linear and recursive  
D. Main and subordinate

You'll notice that this example also contains the phrase “which of the following.” This phrase helps you determine that your answer will be a “relationship of ideas” from the options provided. You are supposed to find the option that describes how entries, or ideas, in outlines are related.

Sometimes it helps to put the question in your own words. Here, you could paraphrase the question in this way: “How are outlines usually organized?” Since the ideas in outlines usually appear as main ideas and subordinate ideas, the answer is D.

**QUICK TIP ➔** Don’t be intimidated by words you may not understand. It might be easy to be thrown by words like “recursive” or “inferential.” Read carefully to understand the question and look for an answer that fits. An outline is something you are probably familiar with and expect to teach to your students. Remember to slow down, and use what you know.

**Watch out for selected-response questions containing “NOT,” “LEAST,” and “EXCEPT”**

This type of question asks you to select the option that does not fit. You must be very careful because it is easy to forget that you are selecting the negative. This question type is used in situations in which there are several good solutions or ways to approach something, but also a clearly wrong way.

**How to approach questions about graphs, tables, or reading passages**

Some questions include introductory information such as a map, table, graph, or reading passage (often called a stimulus) that provides the information the question asks for. In the case of a map or graph, you might want to read the question first, and then look at the map or graph. In the case of a long reading passage, you might want to read the passage first, make notes about places you think are important, and then review your notes and answer the question.

You may also encounter several questions that relate to a single table, graph, or reading passage. There may also be a group of questions that has an initial stimulus that sets the scene and provides information, with a second stimulus at some later point in the questions that provides more information or a subsequent development.

The important thing is to be sure you answer the questions as they refer to the material presented. Be sure to read each question carefully.
How to approach other objective question formats

New formats for presenting information are developed from time to time. Tests may include audio and video stimulus materials such as a movie clip or some kind of animation, instead of a map or reading passage. Other tests may allow you to zoom in on the details in a graphic or picture. Pay attention to the directions on each screen to be sure you understand how the information is being presented for each question or group of questions.

Tests may also include interactive types of questions. These questions take advantage of technology to assess knowledge and skills that go beyond what can be assessed using standard single-selection selected-response questions. If you see a format you are not familiar with, read the directions carefully. The directions always give clear instructions on how you are expected to respond.

The interactive question types may ask you to respond by:

• **Typing in an entry box.** When the answer is a number, you might be asked to enter a numeric answer or, if the test has an on-screen calculator, you might need to transfer the calculated result from the calculator into the entry box. Some questions may have more than one place to enter a response.

• **Clicking check boxes.** You may be asked to click check boxes instead of an oval when more than one option within a set of answers can be selected.

• **Clicking parts of a graphic.** In some questions, you will choose your answer by clicking on location(s) on a graphic such as a map or chart, as opposed to choosing from a list.

• **Clicking on sentences.** In questions with reading passages, you may be asked to choose your answer by clicking on a sentence or sentences within the reading passage.

• **Dragging and dropping answer options into “targets” on the screen.** You may be asked to choose an answer from a list and drag it into the appropriate location in a table, paragraph of text, or graphic.

• **Selecting options from a drop-down menu.** This type of question will ask you to select the appropriate answer or answers by selecting options from a drop-down menu (e.g., to complete a sentence).

Remember that with every question, you will get clear instructions on how to respond. See the GACE Computer-delivered Testing Demonstration on the GACE website to learn how a GACE test works and see examples of some of the types of questions you may encounter.

**QUICK TIP ➔** Don’t make the questions more difficult than they are. Don’t read for “hidden meanings” or “tricks.” There are no “trick questions” on the GACE assessments. They are intended to be serious, straightforward tests of your knowledge.
Understanding Constructed-response Questions

Constructed-response questions require you to demonstrate your knowledge in a subject area by providing in-depth explanations on particular topics. Essay, problem-solving, and oral-response are types of constructed-response questions.

For example, an essay or oral-response question might present you with a topic and ask you to discuss the extent to which you agree or disagree with the opinion stated. You must support your position with specific reasons and examples from your own experience, observations, or reading.

Take a look at a few sample essay topics:

- “Celebrities have a tremendous influence on the young, and for that reason, they have a responsibility to act as role models.”
- “We are constantly bombarded by advertisements — on television and radio, in newspapers and magazines, on highway signs, and the sides of buses. They have become too pervasive. It’s time to put limits on advertising.”
- “Advances in computer technology have made the classroom unnecessary, since students and teachers are able to communicate with each other from computer terminals at home or at work.”

Keep these things in mind when you respond to a constructed-response question

1) **Answer the question accurately.** Analyze what each part of the question is asking you to do. If the question asks you to describe or discuss, you should provide more than just a list.

2) **Answer the question completely.** If a question asks you to do three distinct things in your response, you should cover all three things for the best score. No matter how well you respond, you will not be awarded full credit if you do not answer the question completely.

3) **Answer the question that is asked.** Do not change the question or challenge the basis of the question. You will receive no credit or a low score if you answer another question or if you state, for example, that there is no possible answer.

4) **Give a thorough and detailed response.** You must demonstrate that you have a thorough understanding of the subject matter. However, your response should be straightforward and not filled with unnecessary information.

5) **If your response is written, reread it.** Check that you have written what you thought you wrote. Be sure not to leave sentences unfinished or omit clarifying information.

**QUICK TIP ➔** Scratch paper and pencils will be provided at the test center. You may find that it helps to take notes on this scratch paper about each of the details of the question so that you don't miss any of them. Then you'll be sure to have all of the information you need to answer the question.

For more detailed information on constructed-response scoring, see *Understanding Your GACE® Scores* in the Scores section of the GACE website at [www.gace.ets.org](http://www.gace.ets.org).
GACE Scores

Of course, passing the GACE assessments is important to you, so you need to understand what those scores mean and what the Georgia state standards are.

How do I know if I passed?

All GACE test results, with the exception of the Assessment of Sign Communication – American Sign Language (ASC–ASL), are reported as scaled scores with a scale of 100 to 300. The ASC–ASL is assigned ratings of 1 to 5. Your official score report will indicate how you performed on the test as a whole and whether or not you passed.

IMPORTANT NOTE: For assessments composed of more than one test, you must pass all tests for that assessment to meet the certification requirements.

What Your GACE Scores Mean

You received your score report. Now what does it mean? It’s important to interpret your score report correctly and to know what to do if you have questions about your scores.

Visit the Scores section of the GACE website to see a sample score report and to access Understanding Your GACE® Scores, a document that provides additional information on how to read your score report.

Put your scores in perspective

Your score report indicates:

- the date of the test administration
- your scaled score
- pass/not pass determination
- number of scored questions
- number of questions answered correctly
- number of scored questions in each subarea
- number of questions answered correctly in each subarea
- points possible for constructed-response questions (if your test includes a constructed-response section)
- points earned for constructed-response questions
If an assessment consists of more than one test, the following data will be provided if you have ever taken any of the other tests:

- highest score to date on the test (status, scaled score, and date taken)
- passing status based on the highest scaled score for each of the tests

If you have previously taken the same assessment or other assessments, your score report will also list the highest score you earned on each assessment.

**Score scale changes**

ETS updates GACE assessments on a regular basis to ensure they accurately measure the knowledge and skills that are required for certification. Updated assessments cover the same content as the previous assessments. However, scores might be reported on a different scale, so requirements may vary between the new and previous versions. All scores for previous, discontinued assessments are valid and reportable for 50 years.

*Understanding Your GACE® Scores*, found in the Scores section of the GACE website at [www.gace.ets.org](http://www.gace.ets.org), will help you interpret your scores.
Determine Your Strategy for Success

Effective test preparation doesn't just happen. You'll want to set clear goals and deadlines for yourself along the way. Otherwise, you may not feel ready and confident on test day.

1) Learn what the assessment covers
You may have heard that there are several different versions of the same test. It's true. You may take one version of the test and your friend may take a different version. Each test has different questions covering the same subject area, but both versions of the test measure the same skills and content knowledge.

You'll find specific information on the test you’re taking in the About the Assessment section of each Study Companion, which outlines the content areas that the test measures and what percentage of the test covers each area. Visit the GACE website at www.gace.ets.org for information on other GACE assessments.

2) Assess how well you know the content
Research shows that test takers tend to overestimate their preparedness — this is why some test takers assume they did well and then are surprised to find out they did not pass.

The GACE assessments are demanding enough to require serious review of likely content, and the longer you’ve been away from the content the more preparation you will most likely need. If it has been longer than a few months since you’ve studied your content area, make a concerted effort to prepare.

3) Collect study materials
Gathering and organizing your materials for review are critical steps in preparing for the GACE assessments. Consider the following reference sources as you plan your study:

- Did you take a course in which the content area was covered?
- Do you still have your books or your notes?
- Does your college library have a good introductory college-level textbook in this area?
- Does your local library have a high school-level textbook?

Study Companions are available for all GACE assessments in the Test Prep section of the GACE website at www.gace.ets.org. Each Study Companion provides a combination of test preparation and practice, including sample questions and answers with explanations.

4) Plan and organize your time
You can begin to plan and organize your time while you are still collecting materials. Allow yourself plenty of review time to avoid cramming new material at the end. Here are a few tips:

- Choose a test date far enough in the future to leave you plenty of preparation time. See information on test dates on the GACE website at www.gace.ets.org.
- Work backward from that date to figure out how much time you will need for review.
- Set a realistic schedule — and stick to it.

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
5) Practice explaining the key concepts

Those GACE assessments with constructed-response questions assess your ability to explain material effectively. As a teacher, you’ll need to be able to explain concepts and processes to students in a clear, understandable way. What are the major concepts you will be required to teach? Can you explain them in your own words accurately, completely, and clearly? Practice explaining these concepts to test your ability to effectively explain what you know.

6) Understand how questions will be scored

Scoring information can be found in the Scores section of the GACE website at www.gace.ets.org.

7) Develop a study plan

A study plan provides a roadmap to prepare for the GACE assessments. It can help you understand what skills and knowledge are covered on the test and where to focus your attention. Use the blank study plan template in the back of this Study Companion to organize your efforts.

And most importantly — get started!

Would a Study Group Work for You?

Using this Study Companion as part of a study group

People who have a lot of studying to do sometimes find it helpful to form a study group with others who are working toward the same goal. Study groups give members opportunities to ask questions and get detailed answers. In a group, some members usually have a better understanding of certain topics, while others in the group may be better at other topics. As members take turns explaining concepts to each other, everyone builds self-confidence.

If the group encounters a question that none of the members can answer well, the group can go to a teacher or other expert and get answers efficiently. Because study groups schedule regular meetings, members study in a more disciplined fashion. They also gain emotional support. The group should be large enough so that various people can contribute various kinds of knowledge, but small enough so that it stays focused. Often, three to six members is a good size.

Here are some ways to use this Study Companion as part of a study group:

- **Plan the group’s study program.** Parts of the study plan template can help to structure your group’s study program. By filling out the first five columns and sharing the worksheets, everyone will learn more about your group’s mix of abilities and about the resources, such as textbooks, that members can share with the group. In the sixth column (“Date planned to study this content”), you can create an overall schedule for your group’s study program.

- **Plan individual group sessions.** At the end of each session, the group should decide what specific topics will be covered at the next meeting and who will present each topic. Use the content subareas and objectives in the About the Assessment section to select topics, and then select practice questions.
• **Prepare your presentation for the group.** When it’s your turn to present, prepare something that is more than a lecture. Write two or three original questions to pose to the group. Practicing writing actual questions can help you better understand the topics covered on the test as well as the types of questions you will encounter on the test. It will also give other members of the group extra practice at answering questions.

• **Take a practice test together.** The idea of a practice test is to simulate an actual administration of the test, so scheduling a test session with the group will add to the realism and may also help boost everyone’s confidence. Remember, if you take a practice test, allow only the time that will be allotted for that test on your administration day. You can use the questions in this Study Companion for your practice test.

• **Learn from the results of the practice test.** Check each other’s answers. An answer key for the selected-response questions with explanations for the answers is included in this Study Companion. If your test includes constructed-response questions, look at the constructed-response sample questions, which contain sample responses to those types of questions and shows how they were scored. Then try to follow the same guidelines that the test raters use.
  
  − **Be as critical as you can.** You’re not doing your study partner a favor by letting him or her get away with an answer that does not cover all parts of the question adequately.
  
  − **Be specific.** Write comments that are as detailed as the comments about the sample responses. Indicate where and how your study partner is doing an inadequate job of answering the question. Writing notes for your study partner may also help.
  
  − **Be supportive.** Include comments that point out what your study partner got right and that therefore earned them points.

Then plan one or more study sessions based on aspects of the questions on which group members did not perform well. For example, each group member might be responsible for rewriting one paragraph of a response in which someone else did an inadequate job.

Whether you decide to study alone or with a group, remember that the best way to prepare is to have an organized plan. The plan you follow should set goals based on specific topics and skills that you need to learn, and it should commit you to a realistic set of deadlines for meeting these goals. Then you need to discipline yourself to stick with your plan and accomplish your goals on schedule.
Develop Your Study Plan

Developing a study plan helps you prepare for the GACE assessments. A blank study plan worksheet is available in the back of this Study Companion. You can use this worksheet to:

1. **Define Content Areas:** List the most important content areas for your test as defined in About the Assessment beginning on page 26.
2. **Determine Strengths and Weaknesses:** Identify where you have thorough understanding and where you need additional study in each content area.
3. **Identify Resources:** Identify the books, courses, and other resources you plan to use to study for each content area.
4. **Study:** Create and commit to a schedule that provides for regular study periods.

Below is an example of a completed study plan that may help you get started with your own.

**GACE Test Name:** Reading  
**GACE Test Code:** 117  
**I am taking the test on:** October 25, 2014

<table>
<thead>
<tr>
<th>Literal Comprehension</th>
<th>Content covered</th>
<th>Description of content</th>
<th>How well do I know the content? (scale 1–5)</th>
<th>What resources do I have/need for studying this content?</th>
<th>Where can I find the resources I need?</th>
<th>Date planned to study this content</th>
<th>Date completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Ideas</td>
<td>Identify summaries or paraphrases of main idea or primary purpose of reading section</td>
<td>2</td>
<td>Middle school English text book</td>
<td>College library, middle school teacher</td>
<td>9/15/14</td>
<td>9/15/14</td>
<td></td>
</tr>
<tr>
<td>Supporting Ideas</td>
<td>Identify summaries or paraphrases of supporting ideas and specific details in reading selection</td>
<td>2</td>
<td>Middle school English text book</td>
<td>College library, middle school teacher</td>
<td>9/17/14</td>
<td>9/17/14</td>
<td></td>
</tr>
<tr>
<td>Organization</td>
<td>Identify how reading selection is organized in terms of cause/effect and compare/ contrast</td>
<td>3</td>
<td>Middle and high school English text book</td>
<td>College library, middle and high school teachers</td>
<td>9/20/14</td>
<td>9/21/14</td>
<td></td>
</tr>
<tr>
<td>Organization</td>
<td>Identify key transition words/phrases in reading selection and how used</td>
<td>4</td>
<td>Middle and high school English text book</td>
<td>College library, middle and high school teachers</td>
<td>9/25/14</td>
<td>9/26/14</td>
<td></td>
</tr>
<tr>
<td>Vocabulary in Context</td>
<td>Identify meanings of words as used in context of reading selection</td>
<td>3</td>
<td>Middle and high school English text book, dictionary</td>
<td>College library, middle and high school teachers</td>
<td>9/25/14</td>
<td>9/27/14</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.
### Critical and Inferential Comprehension

<table>
<thead>
<tr>
<th>Content covered</th>
<th>Description of content</th>
<th>How well do I know the content? (scale 1–5)</th>
<th>What resources do I have/need for studying this content?</th>
<th>Where can I find the resources I need?</th>
<th>Date planned to study this content</th>
<th>Date completed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Evaluation</strong></td>
<td>Determine whether evidence strengthens, weakens, or is relevant to arguments in reading selection</td>
<td>5</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>10/1/14</td>
<td>10/1/14</td>
</tr>
<tr>
<td><strong>Evaluation</strong></td>
<td>Determine role that an idea, reference, or piece of information plays in author’s discussion/argument</td>
<td>5</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>10/1/14</td>
<td>10/1/14</td>
</tr>
<tr>
<td><strong>Evaluation</strong></td>
<td>Determine if information presented is fact or opinion</td>
<td>4</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>10/1/14</td>
<td>10/1/14</td>
</tr>
<tr>
<td><strong>Evaluation</strong></td>
<td>Identify relationship among ideas presented in reading selection</td>
<td>2</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>10/1/14</td>
<td>10/1/14</td>
</tr>
<tr>
<td><strong>Inferential Reasoning</strong></td>
<td>Determine logical assumptions on which argument or conclusion is based</td>
<td>3</td>
<td>High school textbook, college course notes</td>
<td>College library, middle and high school teachers</td>
<td>10/8/14</td>
<td>10/8/14</td>
</tr>
<tr>
<td><strong>Inferential Reasoning</strong></td>
<td>Determine author’s attitude toward materials discussed in reading selection</td>
<td>2</td>
<td>High school textbook, college course notes</td>
<td>College library, middle and high school teachers</td>
<td>10/8/14</td>
<td>10/8/14</td>
</tr>
<tr>
<td><strong>Inferential Reasoning</strong></td>
<td>Determine author’s attitude toward materials discussed in reading selection</td>
<td>1</td>
<td>High school textbook, college course notes</td>
<td>College library, middle and high school teachers</td>
<td>10/17/14</td>
<td>10/18/14</td>
</tr>
<tr>
<td><strong>Generalization</strong></td>
<td>Recognize or predict ideas/situations that are extensions of, or similar to, what has been presented in reading selection</td>
<td>2</td>
<td>High school textbook, college course notes</td>
<td>College library, middle and high school teachers</td>
<td>10/17/14</td>
<td>10/18/14</td>
</tr>
<tr>
<td><strong>Generalization</strong></td>
<td>Draw conclusions from materials presented in reading selection</td>
<td>3</td>
<td>High school textbook, college course notes</td>
<td>College library, middle and high school teachers</td>
<td>10/23/14</td>
<td>10/23/14</td>
</tr>
<tr>
<td><strong>Generalization</strong></td>
<td>Apply ideas presented in a reading selection to other situations</td>
<td>3</td>
<td>High school textbook, college course notes</td>
<td>College library, middle and high school teachers</td>
<td>10/23/14</td>
<td>10/23/14</td>
</tr>
</tbody>
</table>

**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.
Review Smart Tips for Success

Learn from the experts. Take advantage of these answers to questions you may have and practical tips to help you navigate the GACE assessment and make the best use of your time.

**Should I guess?**

Yes. Your score is based on the number of questions you answer correctly, with no penalty or subtraction for an incorrect answer. When you don’t know the answer to a question, try to eliminate any obviously wrong answers and then guess at the correct one. Try to pace yourself so that you have enough time to carefully consider every question.

**Can I answer the questions in any order?**

Yes. You can go through the questions from beginning to end, as many test takers do, or you can create your own path. Perhaps you will want to answer questions in your strongest area of knowledge first and then move from your strengths to your weaker areas. You can use the “Mark” function to note a question you want to come back to later. There is no right or wrong way; use the approach that works best for you.

**Are there trick questions on the test?**

No. There are no hidden meanings or trick wording. All of the questions on the test ask about subject matter knowledge in a straightforward manner.

**Are there answer patterns on the test?**

No. You might have heard this myth: The answers on selected-response tests follow patterns. Another myth is that there will never be more than two questions with the same lettered answer following each other. Neither myth is true. Select the answer you think is correct based on your knowledge of the subject.

**Can I write on the scratch paper I am given?**

Yes. You can work out problems on the scratch paper provided to you by the test administrator, make notes to yourself, or write anything at all. You may use your scratch paper in any way that is useful to you, but be sure to enter your final answers on the computer. Your scratch paper will be destroyed after you are finished with the assessment.

**Smart Tips for Taking the Test**

1. **Skip the questions you find extremely difficult.** Rather than trying to answer these on your first pass through the test, leave them blank and mark them. Pay attention to the time as you answer the rest of the questions on the test, and try to finish with 10 or 15 minutes remaining so that you can go back over the questions you left blank. Even if you don’t know the answer the second time you read the questions, see if you can narrow down the possible answers, and then guess.

2. **Keep track of the time.** Keep an eye on the timer located in the upper right-hand corner of the computer screen, and be aware of how much time you have left to complete your test. You will probably have plenty of time to answer all of the questions, but if you find yourself becoming stuck on one question, you might decide to move on and return to that question later.

*Note: After clicking on a link, right click and select "Previous View" to go back to original text.*
3. **Read all of the possible answers before selecting one.** Then, reread the question to be sure the answer you have selected really answers the question. Remember, a question that contains a phrase such as “Which of the following does NOT ...” is asking for the one answer that is NOT a correct statement or conclusion.

4. **Check your answers.** If you have extra time left over at the end of the test, look over each question and make sure that you have answered it as you intended. Many test takers make careless mistakes that they could have corrected if they had checked their answers.

5. **Don’t worry about your score when you are taking the test.** No one is expected to answer all of the questions correctly. Your score on this test is not analogous to your score on other similar-looking (but in fact very different!) tests. It doesn’t matter on the GACE assessments whether you score very high or barely pass. If you meet the minimum passing scores along with any other requirements for obtaining teaching certification, you will receive a license. In other words, what matters is meeting the minimum passing score.

6. **Use your energy to take the test, not to get angry at it.** Getting angry at the test only increases stress and decreases the likelihood that you will do your best. Highly qualified educators and test development professionals, all with backgrounds in teaching and educational leadership, worked diligently to make the test a fair and valid measure of your knowledge and skills. The best thing to do is concentrate on answering the questions.
Check on Testing Accommodations

What if I have a disability or health-related need?

If you have a disability or health-related need, you may wish to apply for testing accommodations. ETS is committed to serving test takers with disabilities or health-related needs by providing services and accommodations that are reasonable and appropriate given the purpose of the test. Testing accommodations are available for test takers with disabilities or health-related needs who meet ETS requirements. If you are requesting testing accommodations, you must register by mail or fax through ETS Disability Services and have your accommodations approved before you register to test.

The 2013–14 Bulletin Supplement for Test Takers with Disabilities or Health-related Needs for GACE® assessments contains contact information, procedures for requesting testing accommodations, and registration forms. The Supplement should be used in conjunction with the information in the GACE Registration Bulletin. The Supplement and the Registration Bulletin can both be downloaded free of charge from the Testing Accommodations section of the GACE website at www.gace.ets.org.

Disability documentation policy statements and forms are available through the ETS website at www.ets.org/disabilities/documentation. You should also see Tips for Test Takers with Disabilities at www.ets.org/disabilities/tips.
Do Your Best on Test Day

You followed your study plan. You are ready for the test. Now it’s time to prepare for test day.

Plan to end your review a day or two before the actual test date so you avoid cramming. Take a dry run to the test center so you’re sure of the route, traffic conditions, and parking. Most of all, you want to eliminate any unexpected factors that could distract you from your ultimate goal — passing the GACE assessment!

On the day of the test, you should:

- be well-rested
- wear comfortable clothes and dress in layers
- eat before you take the test to keep your energy level up
- bring valid and acceptable identification with you that contains your name, signature, and photograph
- be prepared to stand in line to check in or to wait while other test takers check in
- select a seat away from doors, aisles, and other high-traffic areas

You can’t control the testing situation, but you can control yourself. Stay calm. Test administrators are well trained and make every effort to provide uniform testing conditions, but don’t let it bother you if the test doesn’t start exactly on time. You will be given the necessary amount of time once it does start.

You can think of preparing for this test as training for an athletic event. Once you’ve trained, prepared, and rested, give it everything you’ve got.

What items am I restricted from bringing into the test center?

You may not bring personal items into the test center such as:

- cell phones, smartphones (e.g., Android™, BlackBerry®, iPhone®), tablets, PDAs, and other electronic, listening, recording, or photographic devices
- handbags, knapsacks, or briefcases
- food or snacks of any kind
- water bottles or canned or bottled beverages
- study materials, books, or notes
- pens, pencils, and scratch paper (the test administrator will provide pencils and scratch paper)
- tobacco
- weapons of any kind

NOTE: All cell phones, smartphones, tablets, PDAs, and other electronic, listening, recording, or photographic devices are strictly prohibited at the test center. If you are found to be in
possession of any of these devices before, during, or after the test administration, your device may be inspected and/or confiscated, and you will be dismissed from the test. Your test scores will be canceled, and you will forfeit your test fees. For more information on what you can bring to the test center, visit the On Test Day section of the GACE website at www.gace.ets.org.

Are You Ready?

Review this list to determine if you’re ready to take your assessment.

- Do you know the Georgia testing requirements for your teaching field?
- Have you followed all of the test registration procedures?
- Do you know the topics that will be covered in each assessment you plan to take?
- Have you reviewed any textbooks, class notes, and course readings that relate to the topics covered?
- Do you know how long the assessment will take and the number of questions it contains?
- Have you considered how you will pace your work?
- Are you familiar with the types of questions that you may encounter during your assessment?
- Are you familiar with the recommended test-taking strategies?
- Have you practiced by working through the practice questions in the Study Companion?
- If constructed-response questions are part of your test, do you understand the scoring criteria for these items?
- If you are repeating a GACE assessment, have you analyzed your previous score report to determine areas where additional study and test preparation could be useful?

If you answered “yes” to the questions above, your preparation has paid off. Now take the GACE assessment, do your best, pass it — and begin your teaching career!
Other Questions You May Have

Here is some supplemental information that can give you a better understanding of the GACE assessments.

What is the purpose of the GACE assessments?
The purpose of the GACE assessments is to assure that candidates have the knowledge and skills needed to perform the job of an educator in Georgia public schools. The GACE assessments are aligned with state and national standards for educator preparation and with state standards for the P–12 student curriculum — the Common Core Georgia Performance Standards (CCGPS) — and the content standards for Georgia’s state-approved educator preparation programs. In other words, each GACE assessment was developed by Georgia educators to measure competency on what is taught in Georgia’s P-12 classrooms.

Who developed the GACE assessments?
Each GACE assessment was developed with diverse representation of Georgia educators from across the state, including the participation of committees of Georgia educators, educator preparation faculty, and other content and assessment specialists. This included individuals from school systems, local schools, institutions of higher education (public and private), and other stakeholders.

What do the GACE assessments measure?
Each GACE assessment consists of one or more tests designed to assess a candidate’s knowledge and skills as required by the guidelines for Georgia educator certification.

Do some GACE assessments have more than one test?
Yes. Some GACE assessments do consist of more than one test. You may take each individual test at separate administrations, or for assessments that offer a combined test format, you may take the combined version at one administration. You must pass all tests within an assessment to achieve certification.

What is certification?
Certification in any area — medicine, law, architecture, accounting, cosmetology, or education — is an assurance to the public that the person holding the certification possesses sufficient knowledge and skills to perform important occupational activities safely and effectively. In the case of teacher certification, a certification tells the public that the individual has met predefined competency standards for beginning teaching practice.

Because certification makes such a serious claim about its holder, certification tests are usually quite demanding. In some fields, certification tests have more than one part and last for more than one day. Candidates for certification in all fields plan intensive study as part of their professional preparation. Some join study groups, while others study alone. Preparing to take a certification test is, in all cases, a professional activity. Because it assesses the entire body of knowledge for the field you are entering, preparing for a certification exam takes planning, discipline, and sustained effort.
How are the assessments updated to ensure the content remains current?

GACE assessments are reviewed regularly. During the first phase of review, ETS conducts an analysis of relevant state and association standards and of the current test content. State certification areas and the results of any relevant job analysis are also considered. If these reviews indicate that the test content needs to be updated, a state advisory committee is convened to develop revised test content specifications. New test questions are then produced following the standard test development methodology.

How long will it take to receive my scores?

Unofficial scores for tests that contain only selected-response questions can be viewed at the conclusion of the test. Official scores for these tests are reported approximately four weeks later.

Score reporting dates for all testing windows can be found in the Scores section of the GACE website at www.gace.ets.org and in the Registration Bulletin.

Can I access my scores online?

Viewing your scores is easy — simply log in to your ETS GACE testing account on the GACE website at www.gace.ets.org and click on your score report.
About the Assessment

<table>
<thead>
<tr>
<th>Assessment Name</th>
<th>Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade Level</td>
<td>6–12</td>
</tr>
<tr>
<td>Test Code</td>
<td>Test I: 022</td>
</tr>
<tr>
<td></td>
<td>Test II: 023</td>
</tr>
<tr>
<td></td>
<td>Combined Test I and Test II: 522</td>
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<td>Testing Time</td>
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<tr>
<td></td>
<td>Test II: 2 hours</td>
</tr>
<tr>
<td></td>
<td>Combined Test I and Test II: 4 hours</td>
</tr>
<tr>
<td>Test Duration</td>
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<tr>
<td></td>
<td>Test II: 2.5 hours</td>
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<tr>
<td></td>
<td>Combined Test I and Test II: 5 hours</td>
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<tr>
<td>Number of Selected-response Questions</td>
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<tr>
<td></td>
<td>Test II: 45</td>
</tr>
<tr>
<td></td>
<td>Combined Test I and Test II: 90</td>
</tr>
<tr>
<td>Test Format</td>
<td>Computer delivered</td>
</tr>
</tbody>
</table>

The GACE Mathematics assessment is designed to measure the professional knowledge of prospective teachers of 6–12 mathematics in the state of Georgia.

This assessment includes two tests. You may take either test individually or the full assessment in a single session. The testing time is the amount of time you will have to answer the questions on the test. Test duration includes time for tutorials and directional screens that may be included in the test.

The questions in this assessment assess both basic knowledge across content areas and the ability to apply principles.

The total number of scored questions is typically lower than the total number of questions on the test. Most tests that contain selected-response questions include embedded pretest questions, which are not used in calculating your score. Including pretest questions in the assessment allows ETS to analyze actual test-taker performance on proposed new questions and determine whether to include them in future versions of the test.
Content Specifications

Each test in this assessment is organized into content subareas. Each subarea is further defined by a set of objectives and their knowledge statements.

- The objectives broadly define what an entry-level educator in this field in Georgia public schools should know and be able to do.
- The knowledge statements describe in greater detail the knowledge and skills eligible for testing.
- Some tests also include content material at the evidence level. This content serves as descriptors of what each knowledge statement encompasses.

The following is a breakdown of the subareas and objectives for the tests in this assessment.
Test I Subareas

<table>
<thead>
<tr>
<th>Subarea</th>
<th>Approx. Percentage of Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Number and Quantity</td>
<td>30%</td>
</tr>
<tr>
<td>II. Algebra</td>
<td>40%</td>
</tr>
<tr>
<td>III. Discrete Mathematics and Calculus</td>
<td>30%</td>
</tr>
</tbody>
</table>

Test I Objectives

Subarea I: Number and Quantity

Objective 1: Understands and applies knowledge of the real number system and vector and matrix quantities

The beginning Mathematics teacher:

A. Understands the properties of exponents
   - Performs operations involving exponents, including negative and rational exponents
   - Demonstrates an understanding of the properties of exponential expressions
   - Uses the properties of exponents to rewrite expressions that have radicals or rational exponents

B. Understands the properties of rational and irrational numbers and the interactions between those sets of numbers
   - Recognizes that the sum or product of two rational numbers is rational
   - Recognizes that the sum of a rational number and an irrational number is irrational
   - Recognizes that the product of a nonzero rational number and an irrational number is irrational
   - Recognizes that the sum or product of two irrational numbers can be rational or irrational

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
C. Is familiar with the representation and modeling of vector quantities and how operations on vectors are performed

- Represents vector quantities by directed line segments and uses appropriate symbols for vectors and their magnitudes
- Finds the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point
- Solves problems involving velocity and other quantities that can be represented by vectors
- Adds vectors end-to-end, component-wise, and by the parallelogram rule
- Given two vectors in magnitude and direction form, determines the magnitude and direction of their sum

D. Understands how to perform operations on matrices and how to use matrices in applications

- Uses matrices to represent and manipulate data
- Multiplies matrices by scalars to produce new matrices
- Adds, subtracts, and multiplies matrices of appropriate dimensions
- Understands that matrix multiplication for square matrices is not a commutative operation but still satisfies the associative and distributive properties
- Understands the role played by zero and identity matrices in matrix addition and multiplication
- Understands that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse

E. Understands how to solve problems involving ratios, proportions, averages, percents, and metric and traditional unit conversions

- Applies the concept of a ratio and uses ratio language and notation to describe a relationship between two quantities and solve problems
- Uses ratio reasoning to convert rates
- Solves problems involving scale factors
- Recognizes and represents proportional and inversely proportional relationships between two quantities
- Uses proportional relationships to solve multistep ratio, average, and percent problems
- Solves measurement and estimation problems involving time, length, temperature, volume, and mass in both the U.S. customary system and the metric system, where appropriate
- Converts units within the metric and customary systems

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
F. Understands various ways to represent, compare, estimate, and perform calculations on very large and very small numbers; e.g., scientific notation, orders of magnitude

- Represents and compares very large and very small numbers
- Uses orders of magnitude to estimate very large and very small numbers
- Performs calculations on numbers in scientific notation

Objective 2: Understands and applies knowledge of quantities and the complex number system

The beginning Mathematics teacher:

A. Understands how to solve problems by reasoning quantitatively; e.g., dimensional analysis, reasonableness of solutions

- Uses units as a way to understand problems and to guide the solution of multistep problems
- Chooses and interprets units consistently in formulas
- Chooses and interprets the scale and the origin in graphs and data displays
- Recognizes the reasonableness of results within the context of a given problem

B. Understands the structure of the natural, integer, rational, real, and complex number systems and how the basic operations (+, –, ×, and ÷) on numbers in these systems are performed

- Solves problems using addition, subtraction, multiplication, and division of rational, irrational, and complex numbers
- Applies the order of operations
- Given operations on a number system, determines whether the properties (e.g., commutative, associative, distributive) hold
- Compares, classifies, and orders real numbers
- Demonstrates an understanding of the properties of counting numbers; e.g., prime, composite, prime factorization, even, odd, factors, multiples

C. Knows how complex numbers and operations on them are represented in the complex plane

- Represents complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers)
- Explains why the rectangular and polar forms of a given complex number represent the same number
- Represents addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane, and uses properties of the representation for computation
- Calculates the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
D. Understands how to work with complex numbers when solving polynomial equations and rewriting polynomial expressions
   - Solves quadratic equations with real coefficients that have complex solutions
   - Extends polynomial identities to the complex numbers; e.g., $x^2 + y^2 = (x + yi)(x - yi)$

E. Knows how to analyze both precision and accuracy in measurement situations
   - Chooses a level of accuracy appropriate to limitations on measurement when reporting quantities
   - Calculates or estimates absolute and relative error in the numerical answer to a problem

Subarea II: Algebra

Objective 1: Sees structure in expressions and understands arithmetic with polynomials and rational expressions

The beginning Mathematics teacher:

A. Understands how to write algebraic expressions in equivalent forms
   - Uses the structure of an expression to identify ways to rewrite it
   - Understands how to rewrite quadratic expressions for specific purposes; e.g., factoring/finding zeros, completing the square/finding maxima or minima
   - Uses the properties of exponents to rewrite expressions for exponential functions

B. Understands how to perform arithmetic operations on polynomials
   - Adds, subtracts, multiplies, and divides polynomials

C. Understands the relationship between zeros of polynomial functions (including their graphical representation) and factors of the related polynomial expressions
   - Knows and applies the remainder theorem: for a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $x - a$ is a factor of $p(x)$
   - Uses factorization to identify zeros of polynomials
   - Uses zeros of a polynomial to construct a rough graph of the function defined by the polynomial

D. Understands how to use the binomial theorem to solve problems
   - Applies the binomial theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n
E. Understands how to rewrite rational expressions and perform arithmetic operations on rational expressions

- Rewrites simple rational expressions in different forms
- Understands that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression
- Adds, subtracts, multiplies, and divides rational expressions

F. Understands the properties of number systems under various operations

- Given operations on algebraic expressions, determines whether properties (e.g., commutative, associative, distributive) hold
- Performs calculations using newly defined functions

Objective 2: Understands how to create equations and how to reason with equations and inequalities

The beginning Mathematics teacher:

A. Understands how to create equations and inequalities that describe relationships

- Creates equations and inequalities in one variable and uses them to solve problems and graph solutions on the number line
- Creates equations and inequalities to represent relationships between quantities, solves problems, and graphs them on the coordinate plane with labels and scales
- Represents constraints by equations, inequalities, or systems of equations and/or inequalities, and interprets solutions as viable or nonviable options in a modeling context
- Rearranges formulas to highlight a quantity of interest; e.g., solve \( d = rt \) for \( t \)

B. Understands how to justify the reasoning process used to solve equations, including analysis of potential extraneous solutions

- States each step in solving a simple equation
- Solves simple rational and radical equations in one variable, incorporating analysis of possible extraneous solutions

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
C. Understands how varied techniques (e.g., graphical, algebraic) are used to solve equations and inequalities

- Solves linear equations and inequalities, including equations with coefficients represented by letters
- Uses the method of completing the square to transform any quadratic equation in \( x \) into the equivalent form \((x - p)^2 = q\)
- Solves equations using a variety of methods (e.g., using graphs, using the quadratic formula, factoring)
- Uses different methods (e.g., discriminant analysis, graphical analysis) to determine the nature of the solutions of a quadratic equation

D. Understands how varied techniques (e.g., graphical, algebraic, matrix) are used to solve systems of equations and inequalities

- Explains why, when solving a system of two equations using the elimination method, replacing one or both equations with a scalar multiple produces a system with the same solutions as the solutions of the original system
- Solves a system consisting of two linear equations in two variables algebraically and graphically
- Solves a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically
- Represents a system of linear equations as a single matrix equation
- Finds the inverse of a matrix if it exists and uses it to solve systems of linear equations
- Explains why the \( x \)-coordinates of the intersection points of the graphs of \( y = f(x) \) and \( y = g(x) \) are the solutions of \( f(x) = g(x) \)
- Finds the solutions of \( f(x) = g(x) \) approximately (e.g., uses technology to graph the functions, makes tables of values, finds successive approximations); includes cases where \( f(x) \) and/or \( g(x) \) are linear, polynomial, rational, absolute value, exponential, or logarithmic functions
- Graphs the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graphs the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes

E. Understands the concept of rate of change of nonlinear functions

- Calculates and interprets the average rate of change of a function presented symbolically, numerically, or graphically over a specified interval
F. Understands the concepts of intercept(s) of a line and slope as a rate of change
  • Calculates and interprets the intercepts of a line
  • Calculates and interprets the slope of a line presented symbolically, numerically, or graphically
  • Estimates the rate of change of a linear function from a graph

G. Understands how to find the zero(s) of functions
  • Uses a variety of techniques to find and analyze the zero(s) (real and complex) of functions

Subarea III: Discrete Mathematics and Calculus

Objective 1: Understands and applies knowledge of discrete mathematics

The beginning Mathematics teacher:

A. Understands sequences; e.g., arithmetic, recursively defined, geometric
  • Writes arithmetic and geometric sequences both recursively and with an explicit formula, uses them to model situations, and translates between the two forms
  • Evaluates, extends, or algebraically represents rules that involve number patterns
  • Explores patterns in order to make conjectures, predictions, or generalizations

B. Understands the differences between discrete and continuous representations (e.g., data, functions) and how each can be used to model various phenomena
  • Understands the differences between discrete and continuous representations; e.g., data, functions
  • Understands how discrete and continuous representations can be used to model various phenomena

C. Knows how to model and solve problems using vertex-edge graphs, trees, and networks
  • Constructs, uses, and interprets simple diagrams to solve problems
  • Solves linear programming problems

D. Understands basic terminology and symbols of logic
  • Understands the basic terminology of logic
  • Uses logic to evaluate the truth of statements
  • Uses logic to evaluate the equivalence of statements; e.g., statement and contrapositive
  • Identifies basic properties of quantifiers; e.g., for all, there exists
  • Negates statements involving quantifiers; e.g., for all, there exists

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
E. Understands how to use counting techniques such as the multiplication principle, permutations, and combinations
- Uses counting techniques to solve problems

F. Understands basic set theory; e.g., unions, differences, and Venn diagrams
- Solves problems using basic set theory; i.e., union, intersection, complement, difference
- Uses Venn diagrams to answer questions about sets

Objective 2: Understands calculus concepts and applies knowledge to solve calculus problems

The beginning Mathematics teacher:

A. Understands the meaning of a limit of a function and how to calculate limits of functions, how to determine when the limit does not exist, and how to solve problems using the properties of limits
- Graphically analyzes the limit of $f(x)$ as $x$ approaches a fixed value from both left and right
- Solves limit problems (e.g., a constant times a function, the sum of two functions, the product and quotient of two functions) using properties of limits, where all limits of the individual functions exist at the value that $x$ is approaching
- Analyzes one-sided limits for various functions to see whether or not the limit exists
- Recognizes limits that do not exist, such as $\lim_{x \to 0} \sin \left( \frac{1}{x} \right)$ and $\lim_{x \to 0} \frac{1}{x^2}$

B. Understands the derivative of a function as a limit, as the slope of a line tangent to a curve, and as a rate of change
- Constructs a function graph for a given function and a given point $(a, f(a))$, and explains what happens to the succession of slopes of secant lines connecting $(a, f(a))$ to $(x, f(x))$ as $x$ approaches $a$, from both the right side and the left side
- Uses the limit definition of the derivative to find the derivative of a given function at a given value of $x$ and to find the derivative function

C. Understands how to show that a particular function is continuous
- Applies the three steps (i.e., $f(a)$ exists, $\lim_{x \to a} f(x)$ exists, and $f(a) = \lim_{x \to a} f(x)$) that are part of the definition of what it means for a function to be continuous at $x = a$ to verify whether a given function is continuous at a given point

D. Knows the relationship between continuity and differentiability
- Gives examples of functions that are continuous at $x = a$ but not differentiable at $x = a$, and explains why
E. Understands how and when to use standard differentiation and integration techniques
   • Uses standard differentiation techniques
   • Uses standard integration techniques
   • Understands the relationship between position, velocity, and acceleration functions of a particle in motion
F. Understands how to analyze the behavior of a function; e.g., extrema, concavity, symmetry
   • Uses the first and second derivatives to analyze the graph of a function
G. Understands how to apply derivatives to solve problems; e.g., related rates, optimization
   • Applies derivatives to solve problems
H. Understands the foundational theorems of calculus; e.g., fundamental theorems of calculus, mean value theorem, intermediate value theorem
   • Solves problems using the foundational theorems of calculus
   • Understands the relationship between differentiation and integration, including the role of the fundamental theorems of calculus
   • Matches graphs of functions with graphs of their derivatives or accumulations
   • Understands how to use differentiation and integration of a function to express rates of change and total change
   • Understands and calculates the average value of a function over an interval; i.e., mean value theorem of integrals
I. Understands how to use integration to compute area, volume, distance, or other accumulation processes
   • Uses integration techniques to compute area, volume, distance, or other accumulation processes
J. Knows how to determine the limits of sequences, if they exist
   • Determines the limits of sequences when they exist
K. Is familiar with simple infinite series
   • Determines if simple infinite series converge or diverge
   • Finds the sum of a simple infinite series if it exists
   • Finds the partial sum of a simple infinite series
   • Models phenomena (e.g., compound interest, annuities, growth, decay) using finite and infinite arithmetic and geometric sequences and series

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
Test II Subareas

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Test II Objectives

Subarea I: Functions

Objective 1: Understands how to interpret functions and apply knowledge to build functions

The beginning Mathematics teacher:

A. Understands the function concept and the use of function notation
   - Understands that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range
   - Uses function notation, evaluates functions, and interprets statements that use function notation in terms of a context
   - Recognizes that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers
   - Determines the domain and range of a function from a function rule (e.g., $f(x) = 2x + 1$), graph, set of ordered pairs, or table

B. Understands how function behavior is analyzed using different representations; e.g., graphs, mappings, tables
   - For a function that models a relationship between two quantities, interprets key features of graphs and tables (e.g., increasing/decreasing, maximum/minimum, periodicity) in terms of the quantities
   - Given a verbal description of a relation, sketches graphs that show key features of that relation
   - Graphs functions (i.e., radical, piecewise, absolute value, polynomial, rational, logarithmic, trigonometric) expressed symbolically, and identifies key features of the graph
   - Writes a function that is defined by an expression in different but equivalent forms to reveal different properties of the function; e.g., zeros, extreme values, symmetry of the graph

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
• Interprets the behavior of exponential functions; e.g., growth, decay
• Understands how to determine if a function is odd, even, or neither, and any resulting symmetries

C. Understands how functions and relations are used to model relationships between quantities
• Writes a function that relates two quantities
• Determines an explicit expression or a recursive process that builds a function from a context

D. Understands how new functions are obtained from existing functions; e.g., compositions, transformations, inverses
• Describes how the graph of \( g(x) \) is related to the graph of \( f(x) \), where
  \[ g(x) = f(x) + k, \quad g(x) = k \, f(x), \quad g(x) = f(kx), \quad \text{or} \quad g(x) = f(x + k) \]
  for specific values of \( k \) (both positive and negative), and finds the value of \( k \) given the graphs
• Determines if a function has an inverse and writes an expression for the inverse
• Verifies by composition if one function is the inverse of another
• Given that a function \( f \) has an inverse, finds values of the inverse function from a graph or a table of \( f \)
• Given a noninvertible function, determines the largest possible domain of the function that produces an invertible function
• Understands the inverse relationship between exponential and logarithmic functions, and uses this relationship to solve problems
• Combines standard function types using arithmetic operations
• Performs domain analysis on functions resulting from arithmetic operations
• Composes functions algebraically, numerically, and graphically
• Performs domain analysis on functions resulting from compositions

Objective 2: Understands and applies knowledge of linear, quadratic, and exponential models and trigonometric functions

The beginning Mathematics teacher:

A. Understands differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems
• Understands that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals
• Recognizes situations in which one quantity changes at a constant rate per unit interval relative to another
• Recognizes situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
• Constructs linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (including reading these from a table)
• Observes that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function
• Expresses the solution to an exponential equation with base \( b \) as a logarithm; e.g., \( 3 \cdot 2^{5t} = 20, \ 3 \cdot e^{5t} = 20 \)
• Uses technology to evaluate logarithms that have any base
• Interprets the parameters in a linear or exponential function in terms of a context; e.g., \( A(t) = Pe^t \)
• Uses quantities that are inversely related to model phenomena

B. Understands how to construct the unit circle and how to use it to find values of trigonometric functions for all angle measures in their domains
• Finds the values of trigonometric functions of any angle
• Uses the unit circle to explain symmetry and periodicity of trigonometric functions

C. Understands how periodic phenomena are modeled using trigonometric functions
• Chooses trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline
• Understands how to restrict the domain of a trigonometric function so that its inverse can be constructed
• Uses inverse functions to solve trigonometric equations that arise in modeling contexts, and interprets them in terms of the context

D. Understands the application of trigonometric identities (e.g., Pythagorean, double angle, half angle, sum of angles, difference of angles)
• Proves Pythagorean identities (e.g., \( \sin^2 \theta + \cos^2 \theta = 1 \)) and uses them to solve problems
• Uses trigonometric identities to rewrite expressions and solve equations
• Understands trigonometric identities in the context of equivalent graphs of trigonometric functions; e.g., \( y = \sin x \) and \( y = \cos \left( \frac{\pi}{2} - x \right) \) are equivalent graphs

E. Knows how to interpret representations of functions of two variables; e.g., three-dimensional graphs, tables
• Interprets representations of functions of two variables; e.g., \( z = f(x, y) \)

F. Understands how to solve trigonometric, logarithmic, and exponential equations
• Solves trigonometric, logarithmic, and exponential equations

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
Subarea II: Geometry

Objective 1: Understands congruence/similarity/triangles/trigonometric ratios and equations for geometric properties

The beginning Mathematics teacher:

A. Understands transformations in a plane
   - Knows precise definitions of angle, circle, line segment, perpendicular lines, and parallel lines
   - Represents transformations in the plane
   - Recognizes whether a transformation preserves distance and angle measure
   - Given a rectangle, parallelogram, trapezoid, or regular polygon, describes the rotations and reflections that map it onto itself
   - Develops definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments
   - Given a geometric figure and a rotation, reflection, or translation, draws the transformed figure
   - Specifies a sequence of transformations that will map a given figure onto another figure

B. Understands how to prove geometric theorems such as those about lines and angles, triangles, and parallelograms
   - Proves theorems about lines and angles
   - Proves theorems about triangles
   - Proves theorems about parallelograms

C. Understands how geometric constructions are made with a variety of tools and methods
   - Recognizes formal geometric constructions
   - Explains how formal geometric constructions are made; e.g., an equilateral triangle, a square, a regular hexagon inscribed in a circle

D. Understands congruence and similarity in terms of transformations
   - Uses geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure
   - Verifies the properties of dilations given by a center and a scale factor
   - Given two figures, uses the definition of congruence in terms of rigid motions to decide if they are congruent
   - Given two figures, uses the definition of similarity in terms of dilations to decide if they are similar

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
• Explains how the criteria for triangle congruence (e.g., ASA, SAS, SSS, HL) follow from the definition of congruence in terms of rigid motions

• Uses the properties of similarity transformations to establish the AA criterion for two triangles to be similar

• Uses congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures

E. Understands how trigonometric ratios are defined in right triangles

• Understands that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles

• Explains and uses the relationship between the sine and cosine of complementary angles

• Uses trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems

F. Understands how trigonometry is applied to general triangles

• Uses the formula \( A = \frac{1}{2}ab\sin(C) \) for the area of a triangle to solve problems

• Applies the Law of Sines and the Law of Cosines to find unknown measurements in triangles

G. Knows how to translate between a geometric description (e.g., focus, asymptotes, directrix) and an equation for a conic section

• Determines and uses the equation of a circle of given center and radius

• Finds the center and radius of a circle given by an equation in standard form

• Determines the equation of a parabola given a focus and directrix

• Determines and uses the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from a point on the curve to the foci is constant

H. Understands how to use coordinate geometry to algebraically prove simple geometric theorems

• Uses coordinates to prove simple geometric theorems algebraically

• Proves the slope criteria for parallel and perpendicular lines, and uses parallel and perpendicular lines to solve geometric problems

• Finds the point on a directed line segment between two given points that partitions the segment in a given ratio

• Uses coordinates to compute perimeters of polygons and areas of triangles and quadrilaterals
Objective 2: Understands circles, geometric measurement and dimension, and modeling with geometry

The beginning Mathematics teacher:

A. Understands and applies theorems about circles
   - Identifies and describes relationships among inscribed angles, radii, and chords
   - Proves properties of angles for a quadrilateral inscribed in a circle
   - Constructs a tangent line from a point outside a given circle to the circle

B. Understands arc length and area measurements of sectors of circles
   - Uses the length of the arc intercepted by a central angle or inscribed angle to solve circumference problems
   - Uses the formula for the area of a sector to solve problems

C. Understands how perimeter, area, surface area, and volume formulas are used to solve problems
   - Uses the perimeter and area of geometric shapes to solve problems
   - Uses the surface area and volume of prisms, cylinders, pyramids, cones, and spheres to solve problems

D. Knows how to visualize relationships (e.g., cross section, nets, rotations) between two-dimensional and three-dimensional objects
   - Identifies the shapes of two-dimensional cross sections of three-dimensional objects, and identifies three-dimensional objects generated by rotations of two-dimensional objects
   - Uses two-dimensional representations of three-dimensional objects to visualize and solve problems

E. Knows how to apply geometric concepts in real-world situations
   - Uses geometric shapes, their measures, and their properties to describe objects
   - Applies geometric methods to solve design problems

F. Understands the properties of parallel and perpendicular lines, triangles, quadrilaterals, polygons, and circles and how they can be used in problem solving
   - Solves problems involving parallel, perpendicular, and intersecting lines
   - Applies angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
- Solves problems that involve medians, midpoints, and altitudes
- Solves problems involving special triangles; e.g., isosceles, equilateral, right
- Knows geometric properties of and relationships among quadrilaterals; e.g., parallelograms, trapezoids
- Solves problems involving angles and diagonals
- Solves problems involving polygons with more than four sides

**Subarea III: Probability and Statistics**

Objective 1: *Understands how to interpret categorical and quantitative data, make inferences, and justify conclusions*

The beginning Mathematics teacher:

A. Understands how to summarize, represent, and interpret data collected from measurements on a single variable; e.g., box plots, dot plots, normal distributions
   - Represents data with plots on the real number line; e.g., dot plots, histograms, and box plots
   - Uses statistics appropriate to the shape of the data distribution to compare center (e.g., median, mean) and spread (e.g., interquartile range, standard deviation) of two or more different data sets
   - Interprets differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers
   - Uses the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages, and recognizes that there are data sets for which such a procedure is not appropriate

B. Understands how to summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative; e.g., scatterplots, time series
   - Summarizes and interprets categorical data for two categories in two-way frequency tables; e.g., joint, marginal, conditional relative frequencies
   - Recognizes possible associations and trends in the data
   - Represents data for two quantitative variables on a scatterplot, and describes how the variables are related

C. Understands how to create and interpret linear regression models; e.g., rate of change, intercepts, correlation coefficient
   - Uses technology to fit a function to data (i.e., linear regression) and determines a linear correlation coefficient
   - Uses functions fitted to data to solve problems in the context of the data
   - Assesses the fit of a function by plotting and analyzing residuals

**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.
• Interprets the slope and the intercept of a regression line in the context of the data
• Interprets a linear correlation coefficient
• Distinguishes between correlation and causation

D. Understands statistical processes and how to evaluate them
• Understands statistics as a process for making inferences about population parameters based on a random sample from that population
• Decides if a specified model is consistent with results from a given data-generating process; e.g., using simulation

E. Understands how to make inferences and justify conclusions from samples, experiments, and observational studies
• Recognizes the purposes of and differences among sample surveys, experiments, and observational studies, and explains how randomization relates to each
• Uses data from a sample survey to estimate a population mean or proportion
• Uses data from a randomized experiment to compare two treatments
• Uses results of simulations to decide if differences between parameters are significant
• Evaluates reports based on data

Objective 2: Understands conditional probability, the rules of probability, and using probability to make decisions

The beginning Mathematics teacher:

A. Understands the concepts of independence and conditional probability and how to apply these concepts to data
• Describes events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events
• Understands that two events, A and B, are independent if and only if
  \[ P(A \cap B) = P(A) \cdot P(B) \]
• Understands the conditional probability of A given B as
  \[ P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \]
  and interprets independence of A and B as saying that
  \[ P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B) \]

B. Understands how to compute probabilities of simple events, probabilities of compound events, and conditional probabilities
• Calculates probabilities of simple and compound events

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
• Constructs and interprets two-way frequency tables of data when two categories are associated with each object being classified; uses the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities
• Finds $P(A \mid B)$, and interprets it in terms of a given model
• Applies the addition rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interprets it in terms of a given model
• Applies the general multiplication rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B) = P(B)P(A \mid B)$, and interprets it in terms of a given model
• Calculates probabilities using the binomial probability distribution
C. Knows how to make informed decisions using probabilities and expected values
• Defines a random variable for a quantity of interest by assigning a numerical value to each event in a sample space, and graphs the corresponding probability distribution using the same graphical displays as for data distributions
• Calculates the expected value of a random variable, and interprets it as the mean of the probability distribution
• Develops a probability distribution for a random variable, defined for a sample space in which theoretical probabilities can be calculated, and finds the expected value
• Develops a probability distribution for a random variable, defined for a sample space in which probabilities are assigned empirically, and finds the expected value
• Weights the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values
• Analyzes decisions and strategies using probability concepts; e.g., fairness
D. Understands how to use simulations to construct experimental probability distributions and to make informal inferences about theoretical probability distributions
• Given the results of simulations, constructs experimental probability distributions
• Given the results of simulations, makes informal inferences about theoretical probability distributions
E. Understands how to find probabilities involving finite sample spaces and independent trials
• Uses the fundamental counting principle to find probabilities involving finite sample spaces and independent trials

Note: After clicking on a link, right click and select "Previous View" to go back to original text.
Approaches to Answering Selected-response Questions

The purpose of this section is to describe selected-response question formats that you will typically see on the GACE assessments and to suggest possible ways to approach thinking about and answering them. These approaches are intended to supplement and complement familiar test-taking strategies with which you may already be comfortable and that work for you. Fundamentally, the most important component in assuring your success is knowing the content that is covered on the assessment. This content has been carefully selected to align with the knowledge required to begin a career as a beginning teacher in the state of Georgia.

The questions on this assessment are designed to assess your knowledge of the content described in the subareas and objectives in each test. In most cases, you are expected to demonstrate more than just your ability to recall factual information. You may be asked to think critically about the information, to analyze it, to consider it carefully, to compare it with other knowledge you have, or to make a judgment about it.

The questions on this assessment are all selected-response questions. When you are ready to respond, you must choose one of the answer options listed. You may also encounter some questions that use alternate response types; e.g., questions that require you to select multiple options, enter a numeric answer into a text box, or drag-and-drop options. Be sure to read the directions carefully to ensure you know what is required for each test question. Leave no questions unanswered. Questions for which you mark no answer are counted as incorrect. Your score will be determined by the number of questions for which you select the correct answer.

This test has Notations, Definitions, and Formulas built into the testing software. This reference document can be accessed by selecting the “Help” function. The test clock does not stop when the “Help” function is being used.

Note: After clicking on a link, right click and select “Previous View” to go back to original text.
**Question Formats**

You may see the following types of questions on the test:

- Single Questions
- Clustered Questions

On the following pages, you will find descriptions of these commonly used question formats, along with suggested approaches for responding to each type.

**Single Questions**

The single-question format presents a direct question or an incomplete statement. It can also include a reading passage, graphic, table, or a combination of these. The answer options appear below the question.

The following question is an example of the single-question format.

**Example**

Which of the following is the most important consideration for students and teachers with regard to students’ use of the Internet as a research tool?

A. The name of a website does not always give a clear indication of the contents of the site
B. The rapid expansion of the Internet makes it difficult to obtain the very latest information on a given topic
C. Different search engines use different formulas for matching websites to search strings
D. Much of the information on the Internet has not been reviewed and verified by experts in relevant fields

**Suggested Approach**

Read the question carefully and critically. Think about what it is asking and the situation it is describing. Eliminate any obviously wrong answers, select the correct answer choice, and mark your answer.

This question addresses students’ use of the Internet as a research tool. Since there are few controls over what information may be posted on the Internet and by whom, information obtained through this medium cannot be assumed to be accurate. Therefore, students who are using the Internet as a research tool must be made aware of the importance of consulting sources that have been reviewed by experts to verify the accuracy of any information obtained. Therefore, **option D is the single best answer.**
With regard to the other responses, it is true that the name of a website may not accurately represent the information it presents (option A), and it is also true that search engines use different formulas for matching websites to search strings (option C). While these issues may affect how easy it is to find information, they are not relevant to the more critical issue of accuracy. With regard to option B, the question of whether students have located the very latest information, which may or may not be substantiated, is less important than whether they have consulted a variety of up-to-date, accurate resources in a variety of media.

Clustered Questions

Clustered questions are made up of a stimulus and two or more questions relating to the stimulus. The stimulus material may be a reading passage, sample of student work, description of a student and/or program, graphic, table, or any other information necessary to answer the questions that follow.

You can use several different approaches to respond to clustered questions. Some commonly used strategies are listed below.

Strategy 1  Skim the stimulus material to understand its purpose, its arrangement, and/or its content. Then read the questions and refer again to the stimulus material to obtain the specific information you need to answer the questions.

Strategy 2  Read the questions before considering the stimulus material. The theory behind this strategy is that the content of the questions will help you identify the purpose of the stimulus material and locate the information you need to answer the questions.

Strategy 3  Use a combination of both strategies. Apply the “read the stimulus first” strategy with shorter, more familiar stimuli and the “read the questions first” strategy with longer, more complex, or less familiar stimuli. You can experiment with the sample questions in this Study Companion and then use the strategy with which you are most comfortable when you take the actual test.

Whether you read the stimulus before or after you read the questions, you should read it carefully and critically. You may want to note its important points to help you answer the questions.

As you consider questions set in educational contexts, try to enter into the identified teacher’s frame of mind and use that teacher’s point of view to answer the questions that accompany the stimulus. Be sure to consider the questions only in terms of the information provided in the stimulus — not in terms of your own experiences or individuals you may have known.
Example
First read the stimulus (a description of a class activity planned by a teacher).

Use the information below to answer the questions that follow.

A science teacher and a computer teacher teach the same group of eighth graders. The teachers will be addressing some related content with these students, so they agree to create an interdisciplinary unit with coordinated instruction between the two classes.

Now you are prepared to address the first of the two questions associated with this stimulus.

1. The teachers begin planning the interdisciplinary unit by deciding on student learning goals that both teachers will emphasize in their classrooms. To help ensure an effective, well-coordinated unit, the teachers should also decide about which of the following before the unit begins?

A. How to sequence and pace topic coverage during the unit
B. How much time to devote to individual, small-group, and whole-class instruction during the unit
C. What methods to use to communicate with parents and students about learning expectations
D. What presentation methods and teaching styles to use during the unit

Suggested Approach
Read the question carefully and critically. Think about the question that is being asked. Eliminate any obviously wrong answers, select the correct answer choice, and mark your answer.

This question tests understanding of effective collaborative practices. The teachers have agreed on their learning goals for students. Now they need to agree on the sequence and pace of instruction (option A) so that students will be able to build on previously presented content as new content is taught. Students will be much more likely to understand the topic of any given lesson if they are able to fit the new information into a framework of existing knowledge. Therefore, option A is the single best answer.

None of the other responses addresses ways to coordinate instruction effectively. Once the teachers have agreed on student learning goals and on the sequence and pace of topic coverage, students will be able to benefit from instruction whether or not the teachers coordinate their grouping practices (option B) or employ similar presentation methods and teaching styles (option D). With regard to option C, although it is important for each teacher to determine mastery criteria for his or her own subject-area objectives, coordination of instruction during the unit will not depend on agreement between the teachers in regard to the specific criteria each has identified.
Now you are ready to answer the second question.

2. The teachers wish to ensure that their unit will proceed smoothly. They are most likely to achieve this goal by using which of the following strategies?

A. Before the unit begins, create a plan specifying the learning activities that will occur in each teacher’s classroom each day
B. Make arrangements to meet on a regular basis to discuss how the unit is progressing and to address any issues that may arise
C. Before the unit begins, identify any teacher tasks that will need to be done during the unit and assign each task to a teacher
D. Make arrangements to work together to create all of the lesson plans that will be used in both classrooms throughout the unit

**Suggested Approach**

Again, carefully consider the information presented in the stimulus, and then read the second question, which focuses on the principles of effective collaboration in an interdisciplinary teaching situation. Ongoing communication is essential so that the teachers can share information about and identify ways to address such issues as unanticipated directions students’ interests have taken, concepts students are having trouble with, and so forth. Establishing a regular meeting schedule to discuss progress and make necessary adjustments (option B) is an effective means of ensuring that such communication will occur. Therefore, **option B is the single best answer.**

None of the other strategies listed would facilitate the ongoing exchange of information necessary to address issues that arise as the unit proceeds. Since teachers are unlikely to be able to accurately predict the specific issues that will arise during a unit, brainstorming teacher responses before the unit begins (option A) would probably be a very inefficient use of planning time. Jointly creating all lesson plans in advance (option D) would not allow the flexibility necessary to adapt activities and lessons to changing circumstances and would also require a large and unnecessary investment of the teachers’ time. Identifying and assigning specific teacher roles ahead of time (option C), while it might help increase efficiency, would not enhance the teachers’ ability to address student learning issues effectively as they arise.
Practice Questions

This section presents some sample questions for you to review as part of your preparation for the assessment. You will probably find it helpful to simulate actual testing conditions. For each sample test question, there is a correct answer and a rationale.

Keep in mind that the test you take at an actual administration will have different questions, although the proportion of questions in each subarea will be approximately the same. You should not expect the percentage of questions you answer correctly in these practice questions to be exactly the same as when you take the test at an actual administration, since numerous factors affect a person’s performance in any given testing situation.

The sample questions are included to illustrate some of the formats and types of questions you will see on the test; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual test.

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### NOTATIONS PAGE

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**gcd\((m, n)\)**
- greatest common divisor of two integers \(m\) and \(n\)

**lcm\((m, n)\)**
- least common multiple of two integers \(m\) and \(n\)

**\([x]\)**
- greatest integer \(m\) such that \(m \leq x\)

**\(m \equiv k \pmod{n}\)**
- \(m\) and \(k\) are congruent modulo \(n\) (\(m\) and \(k\) have the same remainder when divided by \(n\), or equivalently, \(m - k\) is a multiple of \(n\))

**\(f^{-1}\)**
- inverse of an invertible function \(f\); (not to be read as \(\frac{1}{f}\))

**\(\lim_{x \to a^+} f(x)\)**
- right-hand limit of \(f(x)\); limit (if it exists) of \(f(x)\) as \(x\) approaches \(a\) from the right

**\(\lim_{x \to a^-} f(x)\)**
- left-hand limit of \(f(x)\); limit (if it exists) of \(f(x)\) as \(x\) approaches \(a\) from the left

\(\emptyset\)
- the empty set

**\(x \in S\)**
- \(x\) is an element of set \(S\)

**\(S \subseteq T\)**
- set \(S\) is a proper subset of set \(T\)

**\(S \subseteq T\)**
- either set \(S\) is a proper subset of set \(T\) or \(S = T\)

**\(\overline{S}\)**
- complement of set \(S\); the set of all elements not in \(S\) that are in some specified universal set

**\(T \setminus S\)**
- relative complement of set \(S\) in set \(T\); i.e., the set of all elements of \(T\) that are not elements of \(S\)

**\(S \cup T\)**
- union of sets \(S\) and \(T\)

**\(S \cap T\)**
- intersection of sets \(S\) and \(T\)

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**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.
DEFINITIONS

Discrete Mathematics

A relation $\mathcal{R}$ on a set $S$ is

reflexive if $x \mathcal{R} x$ for all $x \in S$

symmetric if $x \mathcal{R} y \Rightarrow y \mathcal{R} x$ for all $x, y \in S$

transitive if $(x \mathcal{R} y$ and $y \mathcal{R} z) \Rightarrow x \mathcal{R} z$ for all $x, y, z \in S$

antisymmetric if $(x \mathcal{R} y$ and $y \mathcal{R} x) \Rightarrow x = y$ for all $x, y \in S$

An equivalence relation is a reflexive, symmetric, and transitive relation.

FORMULAS

Sum

$\sin (x + y) = \sin x \cos y + \cos x \sin y$
$\cos (x + y) = \cos x \cos y - \sin x \sin y$

Half-Angle (sign depends on the quadrant of $\frac{\theta}{2}$)

$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$
$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$

Range of inverse trigonometric functions

$\sin^{-1} x \quad [-\pi/2, \pi/2]$
$\cos^{-1} x \quad [0, \pi]$
$\tan^{-1} x \quad (-\pi/2, \pi/2)$
Law of Sines

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

Law of Cosines

\[c^2 = a^2 + b^2 - 2ab \cos C\]

DeMoivre’s Theorem

\[(\cos \theta + i \sin \theta)^k = \cos(k \theta) + i \sin(k \theta)\]

Coordinate Transformation

Rectangular \((x, y)\) to polar \((r, \theta)\): \(r^2 = x^2 + y^2\); \(\tan \theta = \frac{y}{x}\) if \(x \neq 0\)

Polar \((r, \theta)\) to rectangular \((x, y)\): \(x = r \cos \theta\); \(y = r \sin \theta\)

Distance from point \((x_1, y_1)\) to line \(Ax + By + C = 0\)

\[d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}\]

Volume

Sphere: radius \(r\)

\[V = \frac{4}{3} \pi r^3\]

Right circular cone: height \(h\), base of radius \(r\)

\[V = \frac{1}{3} \pi r^2 h\]

Right circular cylinder: height \(h\), base of radius \(r\)

\[V = \pi r^2 h\]

Pyramid: height \(h\), base of area \(B\)

\[V = \frac{1}{3} Bh\]

Right prism: height \(h\), base of area \(B\)

\[V = Bh\]
Surface Area

Sphere: radius $r$ 
$$A = 4\pi r^2$$

Lateral surface area of right circular cone: radius $r$, slant height $s$ 
$$A = \pi rs$$

Differentiation

\[
\begin{align*}
(f(x)g(x))' &= f'(x)g(x) + f(x)g'(x) \\
(f'(g(x)))' &= f'(g(x))g'(x) \\
\left( \frac{f(x)}{g(x)} \right)' &= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \quad \text{if } g(x) \neq 0
\end{align*}
\]

Integration by Parts

\[
\int u \, dv = uv - \int v \, du
\]
Directions: Each of the questions or incomplete statements below is followed by four suggested answers or completions. Select the one that is best in each case.

1. The orthogonal projection of 3-space onto the xy-plane takes the point \((x, y, z)\) onto the point \((x, y, 0)\). This transformation can be represented by the matrix equation \(M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}\),

where \(M\) is which of the following matrices?

A. \[
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

B. \[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

C. \[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

D. \[
\begin{pmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

Answer and Rationale
2. For what value of \( x \) is the matrix \( \begin{pmatrix} 1 & 4 \\ x & 6 \end{pmatrix} \) NOT invertible?

A. \( \frac{-3}{2} \)
B. 0
C. \( \frac{3}{2} \)
D. 2

**Answer and Rationale**

3. If \( x \) and \( y \) are even numbers and \( z = 2x^2 + 4y^2 \), then the greatest even number that must be a divisor of \( z \) is

A. 2
B. 4
C. 8
D. 16

**Answer and Rationale**

4. What is the units digit of \( 3^408 \) ?

A. 1
B. 3
C. 7
D. 9

**Answer and Rationale**
5. A taxicab driver charges a fare of $2.00 for the first quarter mile or less and $0.75 for each quarter mile after that. Which of the following equations models the fare, \( f \), in dollars, for a ride \( m \) miles long, where \( m \) is a positive integer?

A. \( f = 2.00 + 0.75(m-1) \)
B. \( f = 2.00 + 0.75 \left( \frac{m}{4} - 1 \right) \)
C. \( f = 2.00 + 0.75(4m-1) \)
D. \( f = 2.00 + 0.75(4(m-1)) \)

Answer and Rationale

6. Given the recursive function defined by \( f(1) = -3 \), \( f(n) = f(n-1) - 6 \), for \( n \geq 2 \), what is the value of \( f(4) \)?

A. \( -2 \)
B. \( -9 \)
C. \( -10 \)
D. \( -21 \)

Answer and Rationale
7. The figure below is a graph of a differentiable function $f$.

Which graph could be the graph of the first derivative of this function?

A. 

B. 

C. 

D. 

Answer and Rationale
8. In a certain chemical reaction, the number of grams, $N$, of a substance produced $t$ hours after the reaction begins is given by $N(t) = 16t - 4t^2$, where $0 < t < 2$. At what rate, in grams per hour, is the substance being produced 30 minutes after the reaction begins?

A. 7  
B. 12  
C. 16  
D. 20

Answer and Rationale

9. If $f(x) = 3x^2$, what are all real values of $a$ and $b$ for which the graph of $g(x) = ax^2 + b$ is below the graph of $f(x)$ for all values of $x$?

A. $a \geq 3$ and $b$ is positive.  
B. $a \leq 3$ and $b$ is negative.  
C. $a$ is negative and $b$ is positive.  
D. $a$ is any real number and $b$ is negative.

Answer and Rationale
10. At the beginning of 2010, the population of rabbits in a wooded area was 250. The function below was used to model the approximate population, \( P \), of rabbits in the area \( t \) years after January 1, 2010.

\[
P(t) = 250 \cdot (3.04)^{\frac{t}{12}}
\]

According to this model, which of the following best describes how the rabbit population changed in the area?

A. The rabbit population doubled every 4 months.
B. The rabbit population tripled every 6 months.
C. The rabbit population doubled every 36 months.
D. The rabbit population tripled every 24 months.

**Answer and Rationale**

11. In \( \triangle ABC \) (not shown), the length of side \( AB \) is 12, the length of side \( BC \) is 9, and the measure of angle \( BAC \) is 30°. What is the length of side \( AC \)?

A. 17.10
B. 4.73
C. 3.68
D. It cannot be determined from the information given.

**Answer and Rationale**
12. For how many angles \( \theta \), where \( 0 < \theta \leq 2\pi \), will rotation about the origin by angle \( \theta \) map the octagon in the figure below onto itself?

![Octagon Diagram](image)

A. One  
B. Two  
C. Four  
D. Eight

**Answer and Rationale**

13. The figure below is not drawn to scale.

![Circle Diagram](image)

In the circle above with center \( O \) and radius 2, tangent \( \overline{AP} \) has length 3 and is tangent to the circle at \( P \). If \( \overline{CP} \) is a diameter of the circle, what is the length of \( \overline{BC} \)?

A. 1.25  
B. 2  
C. 3.2  
D. 5

**Answer and Rationale**
14. The stem-and-leaf plot below shows the course grades that each of 22 students received in a history course. The course grade is represented by using the tens digit of each grade as a stem and the corresponding units digit as a leaf. For example, the stem 9 and the leaf 1 in the first row of the table represent a grade of 91.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
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<tbody>
<tr>
<td>9</td>
<td>1 3 4 5 7</td>
</tr>
<tr>
<td>8</td>
<td>2 2 5 6 6 8 9</td>
</tr>
<tr>
<td>7</td>
<td>0 2 4 5 8 8</td>
</tr>
<tr>
<td>6</td>
<td>1 3 7 9</td>
</tr>
</tbody>
</table>

What is the median course grade of the 22 students?

A. 78
B. 80
C. 80.7
D. 82

Answer and Rationale

15. The measures of the handspans of ninth-grade students at Tyler High School are approximately normally distributed, with a mean of 7 inches and a standard deviation of 1 inch. Of the following, which group is expected to have the greatest percent of measures?

A. The group of handspan measures that are less than 6 inches
B. The group of handspan measures that are greater than 7 inches
C. The group of handspan measures that are between 6 and 8 inches
D. The group of handspan measures that are between 5 and 7 inches

Answer and Rationale
16. A two-sided coin is unfairly weighted so that when it is tossed, the probability that heads will result is twice the probability that tails will result. If the coin is to be tossed 3 separate times, what is the probability that tails will result on exactly 2 of the tosses?

A. \(\frac{2}{9}\)
B. \(\frac{3}{8}\)
C. \(\frac{4}{9}\)
D. \(\frac{2}{3}\)

**Answer and Rationale**
**Answer Key and Rationales**

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| 1               | B              | **Option B is correct.** In order to answer this question, you need to consider how matrix multiplication is performed. You are asked to find a matrix, \( M \), that when multiplied by any matrix of the form \[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\] yields the result \[
\begin{pmatrix}
x \\
y \\
0
\end{pmatrix}
\]. You will notice that all of the answer choices are 3×3 matrices. You can either solve this problem for the general case or reason to the answer. First, the general solution: Let \[
M = \begin{pmatrix}
a & b & c \\
d & e & f \\
g & h & j
\end{pmatrix}
\]. Then \[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
= \begin{pmatrix}
a & b & c \\
d & e & f \\
g & h & j
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix}
ax + by + cz \\
dx + ey + fz \\
gx + hy + jz
\end{pmatrix}
\]
for all \( x \), \( y \), and \( z \). Since \[
\begin{pmatrix}
ax + by + cz = x \\
dx + ey + fz = y \\
gx + hy + jz = 0
\end{pmatrix}
\] This implies \( a = 1, b = 0, c = 0 \); and \( d = 0, e = 1, f = 0 \); and \( g = h = j = 0 \); and, therefore, \[
M = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]. The correct answer is option B. You could also reason to the answer by inspecting the answer choices given. Since multiplying the first row of \( M \) by the matrix \[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\] has to result in only the \( x \) term for all \( x \), \( y \), and \( z \), the first entry in the first row must be 1 and the others 0. Likewise, multiplying the second row of \( M \) by the matrix \[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\] will result only in the \( y \) term for all \( x \), \( y \), and \( z \), so the entries in the second row must be 0, 1, 0, in that order. Multiplying the third row of \( M \) by the... |
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<tr>
<td>Rationale</td>
<td>matrix ( \begin{pmatrix} x \ y \ z \end{pmatrix} ) results in 0 for all ( x ), ( y ), and ( z ), so the entries in the third row must all be 0. Therefore, ( M = \begin{pmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix} ) and the correct answer is option B.</td>
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</table>
| 2               | C             | **Option C is correct.** This question asks you to find the value of $x$ for which the given matrix is NOT invertible. A matrix is not invertible if the determinant of the matrix is equal to zero. The determinant of the matrix \[
\begin{pmatrix}
  a & b \\
  c & d
\end{pmatrix}
\] is equal to $ad - bc$. For the matrix given in the question, the determinant is equal to $(1)(6) - (4)(x)$. This equals 0 when $6 - 4x = 0$, or $x = \frac{3}{2}$. |
| 3               | C             | **Option C is correct.** Since 2 is a divisor of both $2x^2$ and $4y^2$, it follows that 2 is a divisor of $z$. To find out if there is a greater even number that must be a divisor of $z$, you need to consider the additional information given, which is that $x$ and $y$ are both even numbers. Since $x$ and $y$ are even numbers, they can be expressed as $x = 2m$ and $y = 2n$, respectively, where $m$ and $n$ can be either odd or even integers. Substituting these values for $x$ and $y$ into the expression for $z$ yields $z = 2(2m)^2 + 4(2n)^2$. It follows then that $z = 8m^2 + 16n^2$ and that 8 is a divisor of $z$. The number 16 would also be a divisor of $z$ if $m$ is even, but not if $m$ is odd. Since $m$ and $n$ can be either even or odd and the question asks for the largest even number that must be a divisor of $z$, the correct answer is 8, option C. |

**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.
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| 4               | A             | **Option A is correct.** To find the units digit of $33^{408}$, it is helpful to find the first few integer powers of 33 and look for a pattern. For example,  
\[
33^1 = 33 \\
33^2 = 1,089 \\
33^3 = 35,937 \\
33^4 = 1,185,921 \\
33^5 = 39,135,393 \\
\]
You can see that the pattern in the units digit is 3, 9, 7, 1, 3, ... and it will continue to repeat with every four integers of the exponent. Dividing 408 by 4 yields 102 with no remainder. Therefore, the units digit of $33^{408}$ will be the same as the units digit of $33^4$, which is 1. |
<p>| 5               | C             | <strong>Option C is correct.</strong> This question asks you to determine which of the four equations given as choices models the fare for a taxi ride of $m$ miles, where $m$ is a positive integer. The question states that the fare is $2.00 for the first quarter mile or less and $0.75 for each quarter mile after that. You will notice by examining the answer choices that all the choices include a constant term of 2.00 (for the $2.00 for the first quarter mile). Thus, the task is to model the fare for the remaining distance beyond the first quarter mile. Since the question states that $0.75 is charged for each quarter mile after the first, you must determine how many quarter miles there are in the ride. Since the trip is given as $m$ miles (where $m$ is an integer), the number of quarter miles in the trip would be $4m$. The charge for the first quarter mile is $2.00, so that would leave $4m – 1$ quarter miles to be charged at a rate of $0.75 each. The total fare for the trip would thus be modeled by the equation $f = 2.00 + 0.75(4m – 1)$. |</p>
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</table>
| 6               | D              | **Option D is correct.** Given the recursive function defined in the question, to find \( f(4) \), you need first to find \( f(2) \) and \( f(3) \) (\( f(1) \) is given). Since \( f(1) = -3 \) and \( f(n) = f(n - 1) - 6 \) for \( n \geq 2 \), then \[
\begin{align*}
  f(2) &= -3 - 6 = -9 \\
  f(3) &= -9 - 6 = -15 \\
  f(4) &= -15 - 6 = -21
\end{align*}
\] |
| 7               | B              | **Option B is correct.** This question asks you to determine the possible shape of the graph of the first derivative of a differentiable function from the shape of the graph of the function. You should recall that the first derivative of the function at a point is equal to the slope of the graph of the function at that point. By inspection, you will see that, starting near \( x = 0 \), the slope of the graph of \( f(x) \) is negative and becomes less negative as \( x \) approaches \( a \), and that the slope is 0 at \( x = a \) (at the minimum value of \( f \)) and then becomes increasingly positive as \( x \) increases. Only option B is consistent with this behavior. |

**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.
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| 8               | B              | **Option B is correct.** In this question, you are given a function, \( N \), that models the production of a certain chemical reaction in grams as a function of time, \( t \), in hours. You are asked to find the rate of production at 30 minutes after the reaction begins. The rate of production will be equal to the first derivative of \( N \) evaluated at 30 minutes. You should recognize that you first need to convert 30 minutes into hours and then evaluate the first derivative of \( N \) at that value of \( t \). Since 30 minutes equals \( \frac{1}{2} \) hour, you will need to evaluate \( N'\left(\frac{1}{2}\right) \). First find \( N'(t) \).
\[
N'(t) = 16 - 8t.
\]
Therefore, \( N'\left(\frac{1}{2}\right) = 16 - 8\left(\frac{1}{2}\right) = 12 \). The answer is 12 grams per hour. |
| 9               | B              | **Option B is correct.** This question is asking about your understanding of how changing the values of the coefficient \( a \) and the \( y \)-intercept \( b \) in a quadratic function \( f(x) = ax^2 + b \) affects the graph of the function. You should recall that for \( a > 0 \), as \( a \) decreases, the width of the parabola that is the graph of \( y = ax^2 \) increases, and for \( a < 0 \), the graph opens downward. You should also recall that as the value of \( b \) decreases, the vertex of the graph of \( y = ax^2 + b \) moves in a negative direction along the \( y \)-axis.
So for the graph of \( g(x) = ax^2 + b \) to be below the graph of \( f(x) = 3x^2 \) for all values of \( x \), \( a \) must be less than or equal to 3 and \( b \) must be negative (the vertex will be below the vertex of \( f(x) \), which is at the origin). |

**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.
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<td>10</td>
<td>D</td>
<td><strong>Option D is correct.</strong> In this question, a model is given for the growth of the rabbit population as a function of time, ( t ), in years. The question asks for a verbal description of the change in the rabbit population, based on the function given. You should recall the meaning of the base (growth factor) and the exponent in an exponential growth model. You should note that the function given ( P(t) = 250 \cdot (3.04)^{\frac{t}{2}} ) ( \approx 250 \cdot 3^\frac{t}{2} ). You can observe from this approximation (with base 3, and exponent ( \frac{t}{2} )) that the population tripled every two years. Thus the correct answer is option D, “The rabbit population tripled every 24 months.”</td>
</tr>
</tbody>
</table>

**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.
**Question Number** | **Correct Answer** | **Rationale**
--- | --- | ---
11 | D | **Option D is correct.** In this question, you are given the length of two sides of a triangle and the measure of the angle opposite one of those two sides of the triangle. You are asked to find the length of the third side of the triangle. You should recall that the Law of Sines relates the lengths of two sides of a triangle and the sines of the angles opposite the sides. (The Law of Sines is included in the Notations, Definitions, and Formulas pages that are included in this Study Companion and at the beginning of the test.) Using the Law of Sines yields \( \frac{\sin(\angle BAC)}{\sin(\angle BCA)} = \frac{BC}{BA} \) and \( \frac{\sin 30^\circ}{\sin(\angle BCA)} = \frac{9}{12} \).

Therefore, \( \sin(\angle BCA) = \frac{\frac{4}{3} \times \sin 30^\circ}{\frac{2}{3}} = \frac{\frac{4}{3} \times 0.5}{\frac{2}{3}} \). You should recall that this is an example of the ambiguous case of the Law of Sines — that since the value of the sine is between 0 and 1, there are two angles between 0 degrees and 180 degrees, one acute and one obtuse, associated with this sine and therefore there are two possible triangles with the given sides and angle measure. The correct answer, therefore, is (D), “It cannot be determined from the information given.”

The two values of the measure of \( \angle BCA \) are approximately 41.8° and 138.2°. Using either the Law of Sines again (with \( \angle BAC \) and \( \angle ABC \), or with \( \angle BCA \) and \( \angle ABC \)) or the Law of Cosines, you can determine that the length of side \( AC \) is either approximately 3.68 or 17.10. Since the length of side \( AC \) cannot be uniquely determined, the correct answer is option D.

**Back to Question**
### Question 12

**Correct Answer:** B

**Rationale:** Option B is correct. The question asks you to consider rotation about the origin of the octagon in the figure and to determine for how many angles \( \theta \), where \( 0 < \theta \leq 2\pi \), would rotation of the octagon result in the octagon being mapped onto itself. One way to begin is to consider a single point on the octagon, such as the point (0,4), at the top of the octagon in the figure. This point is 4 units from the origin, so any rotation that maps the octagon onto itself would need to map this point onto a point that is also 4 units from the origin. The only other point on the octagon that is 4 units from the origin is the point (0,−4). A rotation of angle \( \theta = \pi \) would map the point (0,4) onto the point (0,−4). You can see that the octagon is symmetric about the x- and y-axes, so a rotation of angle \( \theta = \pi \) would map all of the points of the octagon onto corresponding points of the octagon. Likewise, a rotation of angle \( \theta = 2\pi \) would map the point (0,4) onto itself (and map all other points of the octagon onto themselves). No other values of \( \theta \) such that \( 0 < \theta \leq 2\pi \) would map the octagon onto itself. Therefore, the correct answer is two, option B.

**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.
### Question 13

**Correct Answer:** C

**Rationale:**

Option C is correct. To determine the length of $BC$, it would be helpful to first label the figure with the information given. Since the circle has radius 2, then both $OC$ and $OP$ have length 2 and $CP$ has length 4. $AP$ is tangent to the circle at $P$, so angle $APC$ is a right angle. The length of $AP$ is given as 3. This means that triangle $ACP$ is a 3-4-5 right triangle and $AC$ has length 5. You should also notice that since $CP$ is a diameter of the circle, angle $CBP$ is also a right angle. Angle $BCP$ is in both triangle $ACP$ and triangle $PCB$, and therefore the two triangles are similar. You can then find the length of $BC$ by setting up a proportion between the corresponding parts of the similar triangles as follows:

$$\frac{CP}{AC} = \frac{BC}{PC}$$

$$\frac{4}{5} = \frac{BC}{4}$$

$$BC = \frac{16}{5} = 3.2$$

The correct answer, 3.2, is option C.

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<td>14</td>
<td>D</td>
<td><strong>Option D is correct.</strong> A stem-and-leaf plot such as the one shown in this question is a very useful way to display data such as these when you are interested in determining the median value of the data. The data in a stem-and-leaf plot is ordered, so finding the median, the middle number when the data are ordered from least to greatest or greatest to least, is straightforward. You are given the course grades received by 22 students. The median course grade would be the average of the course grades of the 11th and 12th students. You can start at either the least or greatest data entry and count in increasing (or decreasing) order along the leaves until you reach the 11th and 12th entries. In this case, both the 11th and 12th entries have a value of 82 (i.e., a stem value of 8 and a leaf value of 2). Therefore, the median course grade received by the 22 students is 82.</td>
</tr>
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<tr>
<td>15</td>
<td>C</td>
<td><strong>Option C is correct.</strong> In this question, you will need to use your knowledge of a normally distributed set of data. In particular, you should know that approximately 68 percent of a normally distributed set of data lie within ±1 standard deviation of the mean and approximately 95 percent of the data lie within ±2 standard deviations of the mean. The question asks you to identify which of the groups given in the answer choices is expected to correspond to the greatest percent if the handspan measures are approximately normally distributed with a mean of 7 inches and a standard deviation of 1 inch. You will need to evaluate each answer choice in order to determine which of the groups has the greatest percent. Option A is the group of handspan measures less than 6 inches. Since the mean handspan is 7 inches and the standard deviation is 1 inch, the group of handspan measures that is less than 6 inches is the group that is more than 1 standard deviation less than the mean. The group of handspan measures that is less than 7 inches includes 50 percent of the measures. Approximately 34 percent ((\frac{1}{2}) of 68 percent) of the measures are between 6 inches and 7 inches (within 1 standard deviation less than the mean). So the group with handspan measures less than 6 inches would be approximately equal to 50%–34%, or 16 percent of the measures. Option B is the group of handspan measures greater than 7 inches. Since 7 inches is the mean, approximately 50 percent of the measures are greater than the mean. Option C is the group of handspan measures between 6 and 8 inches. This is the group that is within ±1 standard deviation of the mean. This group contains approximately 68 percent of the measures. Option D is the group of handspan measures between 5 and 7 inches. This group is between the mean and 2 standard deviations less than the mean. Approximately 47.5 percent ((\frac{1}{2}) of 95 percent) of the measures are between 5 inches and 7 inches. Of the options given, the group described in option C is expected to contain the greatest percent of the measures, approximately 68 percent, so option C is the correct answer.</td>
</tr>
</tbody>
</table>

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**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.
**Question Number** | **Correct Answer** | **Rationale**
---|---|---
16 | A | **Option A is correct.** In this question, you are asked to apply your knowledge of independent events to find the probability of tossing tails exactly 2 out of 3 times when using an unfairly weighted coin. Because each toss of the coin is an independent event, the probability of tossing heads and then 2 tails, \( P(HTT) \), is equal to \( P(H) \cdot P(T) \cdot P(T) \), where \( P(H) \) is the probability of tossing heads and \( P(T) \) is the probability of tossing tails. In this case, you are given that the probability of tossing heads is twice the probability of tossing tails. So, \( P(H) = \frac{2}{3} \) and \( P(T) = \frac{1}{3} \). (Out of 3 tosses, 2 would be expected to be heads and 1 would be expected to be tails.) Therefore \( P(HTT) = \left( \frac{2}{3} \right) \cdot \left( \frac{1}{3} \right) \cdot \left( \frac{1}{3} \right) = \frac{2}{27} \). There are 3 ways in which exactly 2 of 3 tosses would be tails, and each of them has an equal probability of occurring: \( P(THT) = P(TTH) = P(HTT) = \frac{2}{27} \). Therefore the total probability that tails will result exactly 2 times in 3 tosses is \( 3 \left( \frac{2}{27} \right) = \frac{2}{9} \). So, the correct answer is option A.

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## Study Plan Sheet

<table>
<thead>
<tr>
<th>Content covered</th>
<th>Description of content</th>
<th>How well do I know the content? (scale 1–5)</th>
<th>What resources do I have/need for studying this content?</th>
<th>Where can I find the resources I need?</th>
<th>Date planned to study this content</th>
<th>Date completed</th>
</tr>
</thead>
<tbody>
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**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.
Preparation Resources

The resources listed below may help you prepare for the GACE assessment in this field. These preparation resources have been identified by content experts in the field to provide up-to-date information that relates to the field in general. You may wish to use current issues or editions to obtain information on specific topics for study and review.

Journals

*Mathematics Teacher*, National Council of Teachers of Mathematics

*Mathematics Teaching in the Middle School*, National Council of Teachers of Mathematics

*Middle Ground*, National Middle School Association

*Middle School Journal*, National Middle School Association

Other Resources


Note: After clicking on a link, right click and select "Previous View" to go back to original text.


**Note:** After clicking on a link, right click and select "Previous View" to go back to original text.


**Online Resources**

Georgia Department of Education — www.doe.k12.ga.us

Mathematics TEKS Toolkit, The Charles A. Dana Center at the University of Texas at Austin — www.utdanacenter.org/mathtoolkit

National Council of Teachers of Mathematics — www.nctm.org